

# EOS of neutron star cores and spin down of pulsars

**Paweł Haensel**

Copernicus Astronomical Center (CAMK)  
Warszawa, Poland  
haensel@camk.edu.pl

*Isolated Neutron Stars*

April 24-28, 2006, London (UK)

*Collaboration with M. Bejger (CAMK Warsaw/LUTH Meudon), J.L. Zdunik (CAMK Warsaw), and E. Gourgoulhon (LUTH Meudon)*

## Motivation

Isolated pulsar loses  $J$  due to radiation. In response to  $\dot{J}$  it changes  $f = 1/\text{period} = \Omega/2\pi$  (observable!) and increases central density and pressure  $\rho_c$ ,  $P_c$ . Response to  $\dot{J}$  depends on the EOS of the neutron-star core, and is sensitive to appearance of new particles or of a new phase. Crossing the phase-transition region by  $\rho_c$  is reflected by specific “nonstandard” behavior of  $f(t)$ .

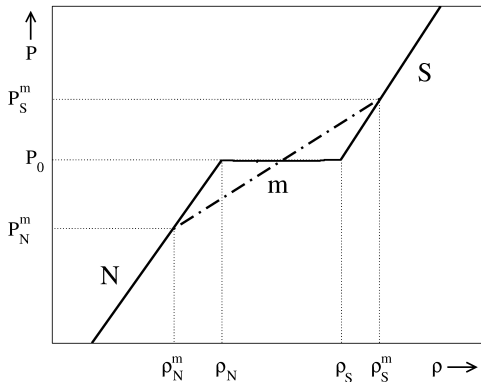
## Plan

- Phase transitions and EOS - full equilibrium
- Back bending and stability
- Stability and rotation
- Instability and corequakes
- Metastability and EOS
- Energy release in a corequake

# 1st order N-S :thermodynamic equilibrium

## pure N-S, density jump

Between two pure phases N and S, at some  $P_0$ , with density jump:  $\rho_N < \rho_S$ . Occurs for sufficiently strong pion or kaon condensation. Characteristic for many models of quark deconfinement.



## pure N - mixed NS - pure S

Above  $P_N^{(m)}$  NS preferred over the pure N, with fraction of the S phase increasing from zero to one at  $P = P_S^{(m)}$ , and above  $P_S^{(m)}$  pure S phase is preferred. Might be possible for: meson condensations or quark matter, provided **surface tension** at the N-S interface is sufficiently small.

# Stability of hydrostatic equilibria

## Stability criteria - nonrotating stars

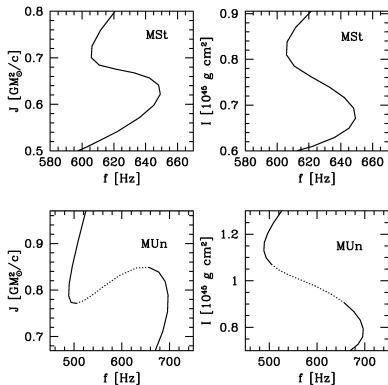
*Wheeler & collaborators (1958)*...

One-parameter family -  
configurations  $\mathcal{C}(x)$ ,  $x = \rho_c, P_c$

radial perturbations:

stable if  $dM/dx > 0$

unstable if  $dM/dx < 0$



## Stability criteria - rotating stars

*Friedman, Ipser, Sorkin (1988)*...

Two-parameter family:  $\mathcal{C}(x, \Omega)$  - **axially symmetric perturbations** - 2-D

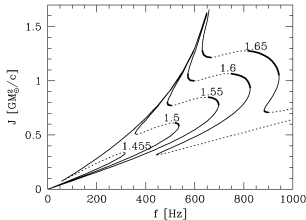
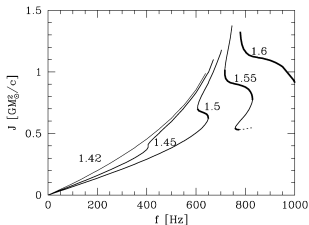
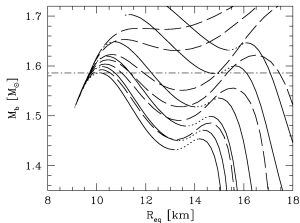
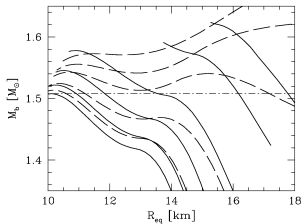
**Criteria I** stable if  $(\partial M/\partial x)_{J=\text{const.}} > 0$     unstable if  $(\partial M/\partial x)_{J=\text{const.}} < 0$

**Criteria II** stable if  $(\partial J/\partial x)_{M_b=\text{const.}} > 0$     unstable if  $(\partial J/\partial x)_{M_b=\text{const.}} < 0$

# Invariance of structure of (one-parameter) families $\{C_X\}$

$$X = M_b, J, f$$

Zdunik, Bejger, Haensel, Gourgoulhon (2005,2006)



stable static  $\implies$  stable rotating

unstable segment static  $\implies$  unstable segment rotating

# Invariance of structure of $\{\mathcal{C}_X\}$ families - continued

$$X = M_b, J, f$$

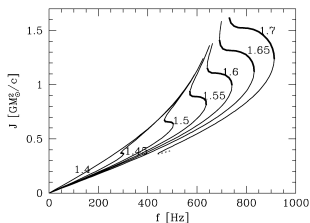
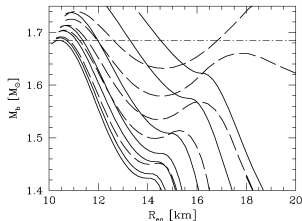
## Marginal instability

Marginally unstable configuration-inflection point  $(\partial M/\partial x)_{J=const.} = 0$  and  $(\partial^2 M/\partial x^2)_{J=const.} = 0$

$(\partial J/\partial x)_{M_b=const.} = 0$  and  $(\partial^2 J/\partial x^2)_{M_b=const.} = 0$

## Conjectures

- I All stable  $\{\mathcal{C}\}_{stat}$  static remains all stable  $\{\mathcal{C}_X\}_{rot}$
- II If  $\{\mathcal{C}\}_{stat}$  contains unstable segment then every  $\{\mathcal{C}_X\}$  contains unstable segment too
- III If  $\{\mathcal{C}\}_{stat}$  contains a marginally unstable  $\mathcal{C}$  then each  $\{\mathcal{C}_X\}$  contains a marginally unstable  $\mathcal{C}_X$



**Conjectures  $\approx$  Theorems because exceptions from them form a set of**

single family of stable static configurations  $\Leftrightarrow$  single family of stable rotating configurations

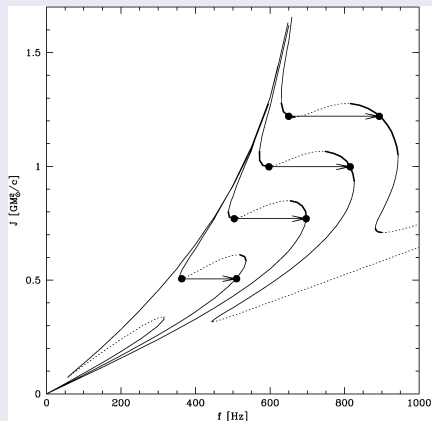
two disjoint families of stable static configurations separated by a family of unstable configurations  $\Leftrightarrow$  two disjoint families of stable rotating configurations separated by a family of unstable configurations (constant  $M_b$ , or constant  $J$ , or constant  $f$ )

generic feature of EOSs with phase transitions, not changed by rigid rotation

# Instability and starquakes

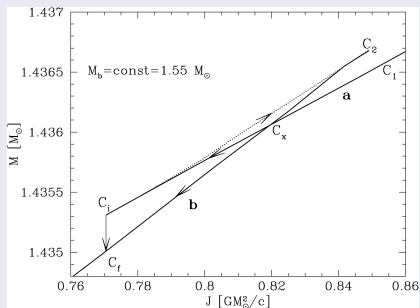
A spinning down pulsar reaches instability point and collapses (**spin up!**) into a new **stable**  $\mathcal{C}$  and then continues its evolution

## Tracks in $J - f$ plane



## Track in $M - J$ plane

The only way out from  $\mathcal{C}_i$  is collapse to  $\mathcal{C}_f$ , keeping  $M_b$  and  $J$  constant. Energy released  $\Delta E = (M_i - M_f)c^2$





# Back bending and pulsar timing

Isolated pulsar energy balance

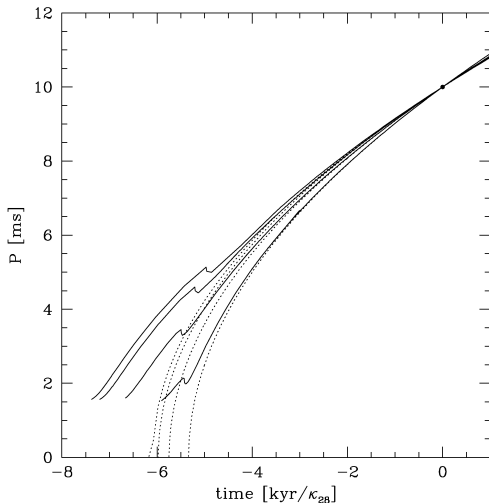
$$\dot{M}c^2 = -\kappa\Omega^\alpha$$

Standard model:  $I = I(0) = \text{const.}$

However, in the phase transition epoch  $I(\Omega)$  is **crucial**. Standard approximation: overestimates spindown rate and underestimates age.

Example: 10 ms pulsar observed at time = 0. Evolution back in time using  $I = \text{const.}$  can be misleading if a phase transition region was crossed  
**huge values of  $n = \Omega\ddot{\Omega}/\dot{\Omega}^2$**   $\longrightarrow$

Period evolution in time



# $\Delta f$ , $\Delta R$ , and $\Delta E$ in starquakes

$$\mathcal{C}_i \longrightarrow \mathcal{C}_f$$

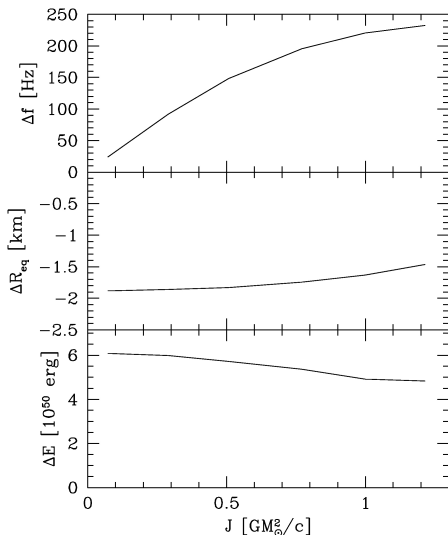
conditions  $M_{b,i} = M_{b,f}$ ,  $J_i = J_f$

Energy release  $\Delta E = -\Delta M c^2$

Characteristic dependence on  $J_i$

Very weak dependence of  $\Delta E$   
on  $J_i$

Therefore  $\Delta E$  can be calculated  
using 1-D code for non-rotating  
stars and this gives excellent  
prediction (within better than  
20%) even for high  $J_i$  ! No need  
for 2-D to get reliable estimate.



# Metastability and two cases of corequakes

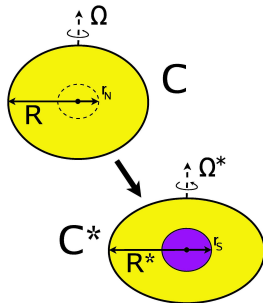
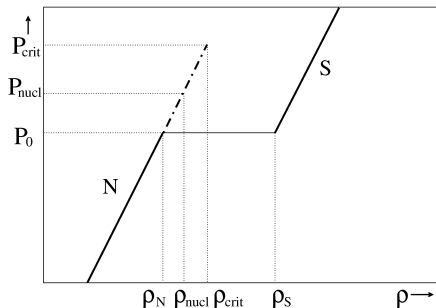
non-rotating stars: *Haensel, Zduńik, Schaeffer (1986,87)*

Two cases of starquakes

(1) weak and moderate 1st order phase transitions  $\rho_S/\rho_N < \frac{3}{2} + P_0/\rho_N c^2$ ;  
starquake due to nucleation of S phase in a metastable core of N phase

(2) strong 1st order phase transitions  $\rho_S/\rho_N > \frac{3}{2} + P_0/\rho_N c^2$  - non-rotating  
configurations with  $P_c > P_0$  with small S cores are unstable (collapse)

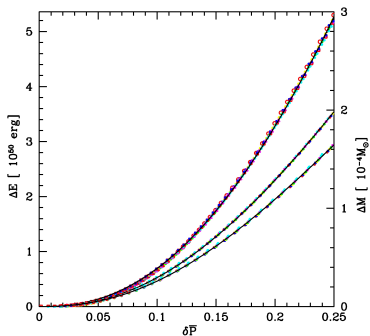
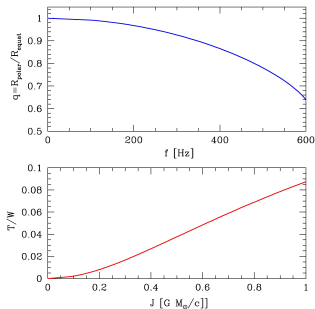
rotating stars: *Zduńik, Bejger, Haensel, Gourgoulhon(2006)*



# $\Delta E$ independent of $J$ !

Overcompression in the N-star center  $\delta\bar{P} \equiv (P_c - P_0)/P_0$ . Starquake triggered for  $P_c = P_{\text{nucl}}$

*Zdunik, Bejger, Haensel, Gourgoulhon (2006)*



**Energy release independent of  $J_i$  also for strong 1st order phase transition. 1-D static calculations are sufficient to get energy release for rotating stars**

*Zdunik, Bejger, Haensel, Gourgoulhon (2006)*