FORCE-FREE MAGNETOSPHERE OF AN ALIGNED ROTATOR

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Pulsar Equation

Force-free magnetosphere of an aligned rotator rotating with the angular velocity Ω can be well described by the so-called pulsar equation

$$(\beta^2x^2-1)(\partial_{xx}\psi+\partial_{zz}\psi)+\frac{\beta^2x^2+1}{x}\partial_x\psi- -S\frac{dS}{d\psi}+x^2\beta\frac{d\beta}{d\psi}(\nabla\psi)^2=0\,. \quad (1)$$

Here all coordinates are normalized to $R_{\rm LC}^{\rm cor}\equiv c/\Omega$, corresponding to the Light Cylinder radius for co-rotating plasma. $\beta\equiv\Omega_F/\Omega$ - is the normalized angular velocity of plasma rotation. We used such normalization for ψ that the dipole magnetic field function near the NS surface is given by

$$\psi^{\rm dip}=\frac{x^2}{(x^2+z^2)^{3/2}}, \quad (2)$$

The normalized poloidal current function $S\equiv(4\pi/c)(R_{\rm LC}^2/\mu)\,I$, $\mu\equiv B_0R_{\rm NS}^3/2$. Magnetic field is expressed through the functions $\Psi\equiv\mu/R_{\rm LC}\,\psi$ and I:

$$\mathbf{B}=\frac{\nabla\Psi\times\mathbf{e}_\phi}{\varpi}+\frac{4\pi}{c}\frac{I}{\varpi}\mathbf{e}_\phi \quad (3)$$

At the Light Cylinder, $R_{\rm LC}\equiv c/\Omega_F$, the pulsar equation has the form

$$2\beta\,\partial_x\psi=S\frac{dS}{d\psi}-\frac{1}{\beta}\frac{d\beta}{d\psi}(\nabla\psi)^2\,. \quad (4)$$

Any *smooth* solution must satisfy this equation at the Light Cylinder (LC). Angular velocity of plasma rotation in the open field line domain is

$$\Omega_{\rm F}=\Omega+c\frac{\partial V}{\partial\Psi}\,. \quad (5)$$

where V is the non-corotational electric potential, caused by presence of an accelerating electric field in the polar cap of pulsar. If from a particularly theory of polar cap cascade we know β as a function of ψ, $\beta=\beta(\psi)$, the position of the Light Cylinder $R_{\rm LC}(x,z)=c/\Omega_F\,[\psi(x,z)]$ must be determined self-consistently together with the solution of the pulsar equation.

Solution method

We assume Y-configuration of the magnetosphere, i.e. the existence of an equatorial current sheet with the return current. Equation (1) is solved numerically in a domain $x_{\rm NS}\le x\le x_{\rm max}$, $z_{\rm NS}\le z\le z_{\rm max}$

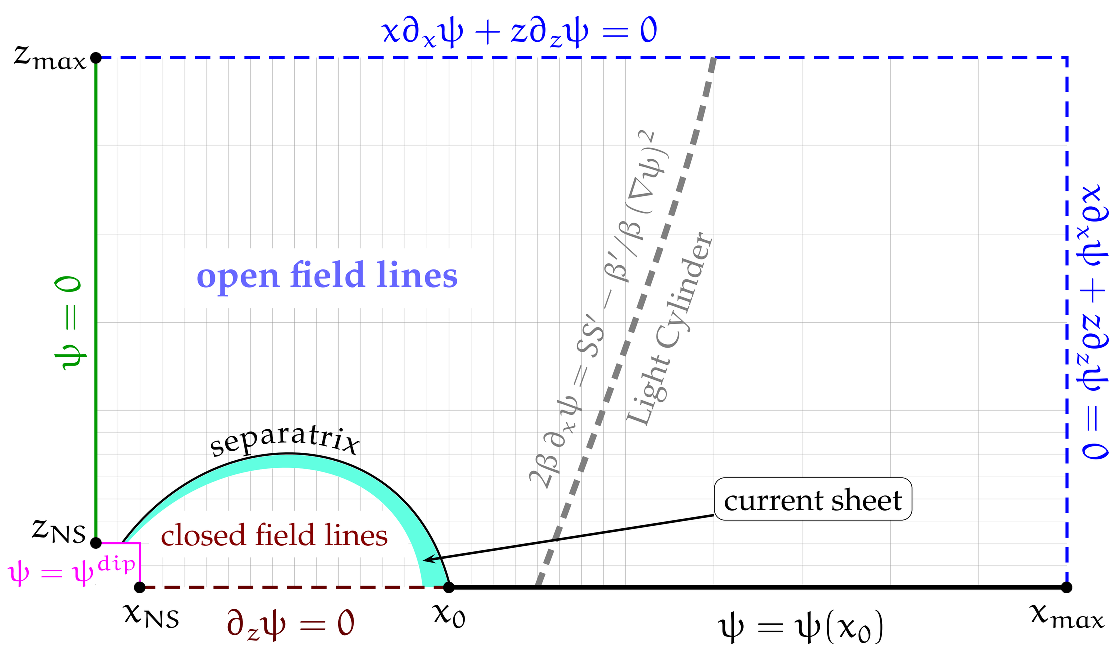

FIGURE 1: *Calculation domain and boundary conditions*

The return current flowing along the last closed magnetic field line is smeared over the region $[\psi_0-d\psi,\psi_0]$.

Boundary conditions are shown in Fig. 1. The equation (1) is solved by a full multigrid (V-cycles) FAS scheme. As a smoother we used Gauss-Seidel scheme.

At each iteration step we find position of the LC $\{(x_{{\rm LC}\,j},z_j),\ j=0\ldots n\}$ by solving numerically the equation

$$x_{{\rm LC}\,j}=1/\beta\left[\psi(x,z_j)\right]\,. \quad (6)$$

by the Newton method for each z-axis grid point z_j, and find poloidal current function $SS'\equiv S\,(dS/d\psi)$ from the equation (4). Then we use piece-polynomial interpolation for SS' and calculate $SS'(x,z)=SS'\,[\psi(x,z)]$ in each domain point.

β in the current sheet smoothly changes to the value $\beta=1$ in the closed field line zone.

References

1. A. N. Timokhin, 2006, MNRAS (in press, doi: 10.1111/j.1365-2966.2006.10192.x), astro-ph/0511817
2. A. N. Timokhin, 2006, in preparation

Main Results

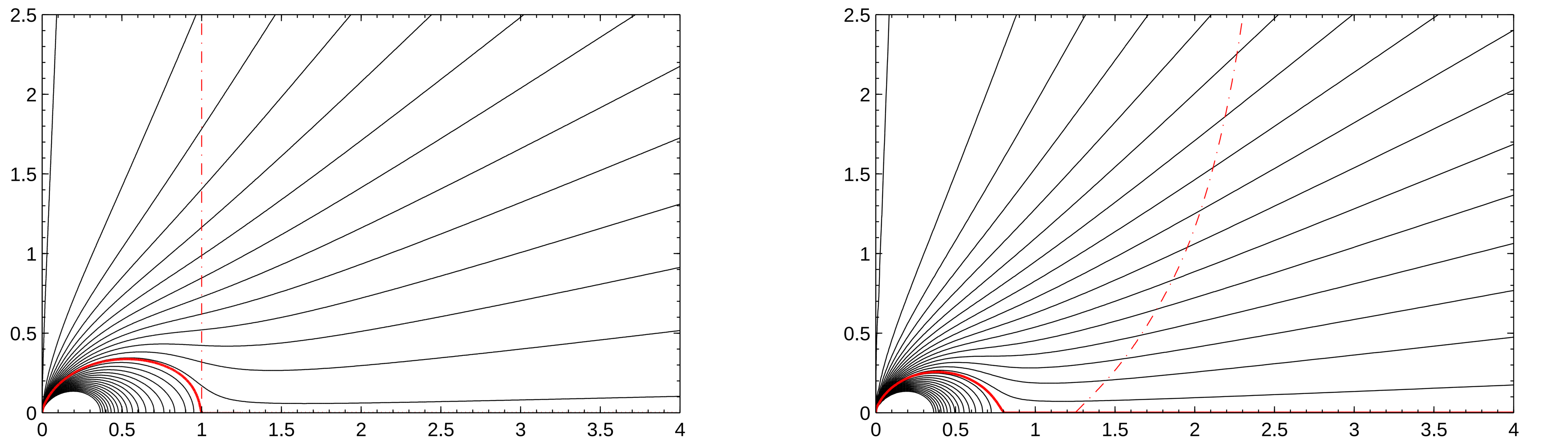

FIGURE 2: *Structure of the pulsar magnetosphere:* ***left*** *– for $\beta\equiv1$ and $x_0=1$;* ***right*** *– for $x_0=0.8$ and β varying in such a way that the current density in the polar cap of pulsar is nearly constant (see Fig.4 (right)). The Light Cylinder is shown by the dot-dashed line.*

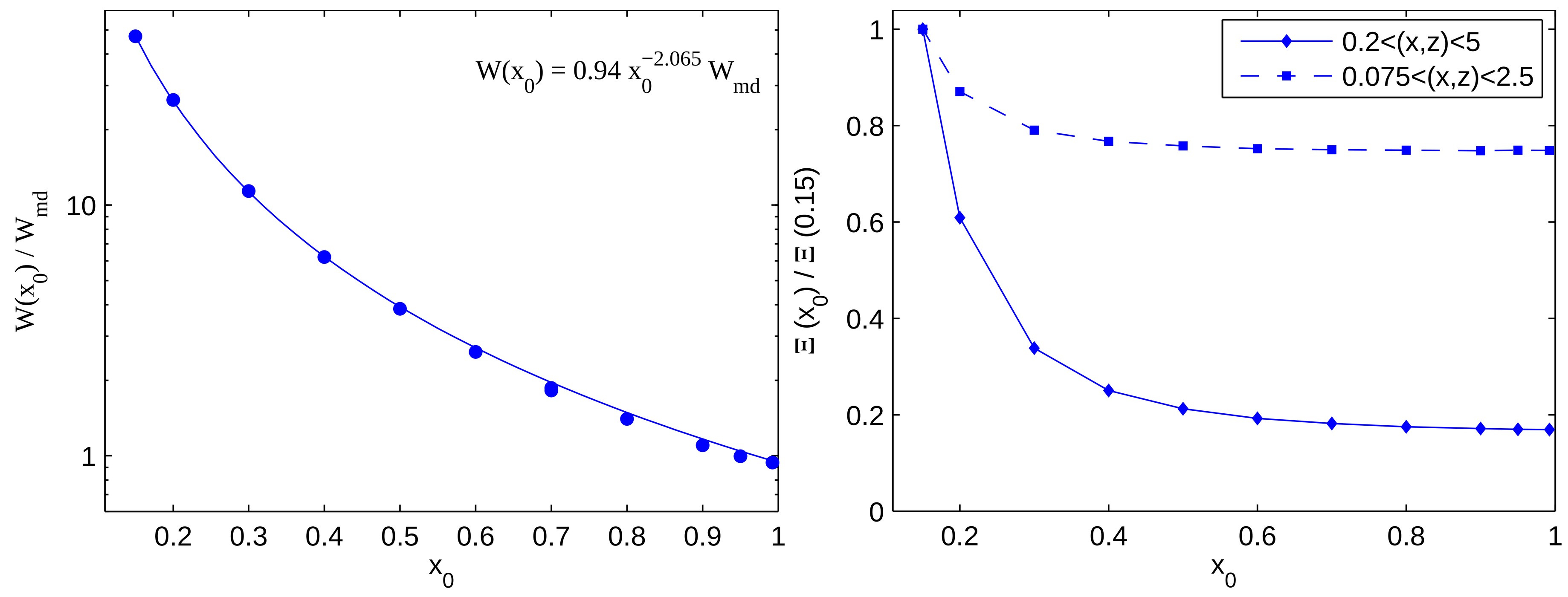

FIGURE 3: *For $\beta\equiv1$:* ***left*** *– energy losses normalized to the magnetodipolar energy losses as a function of x_0;* ***right*** *– total electromagnetic energy in two different volumes as a function of x_0*

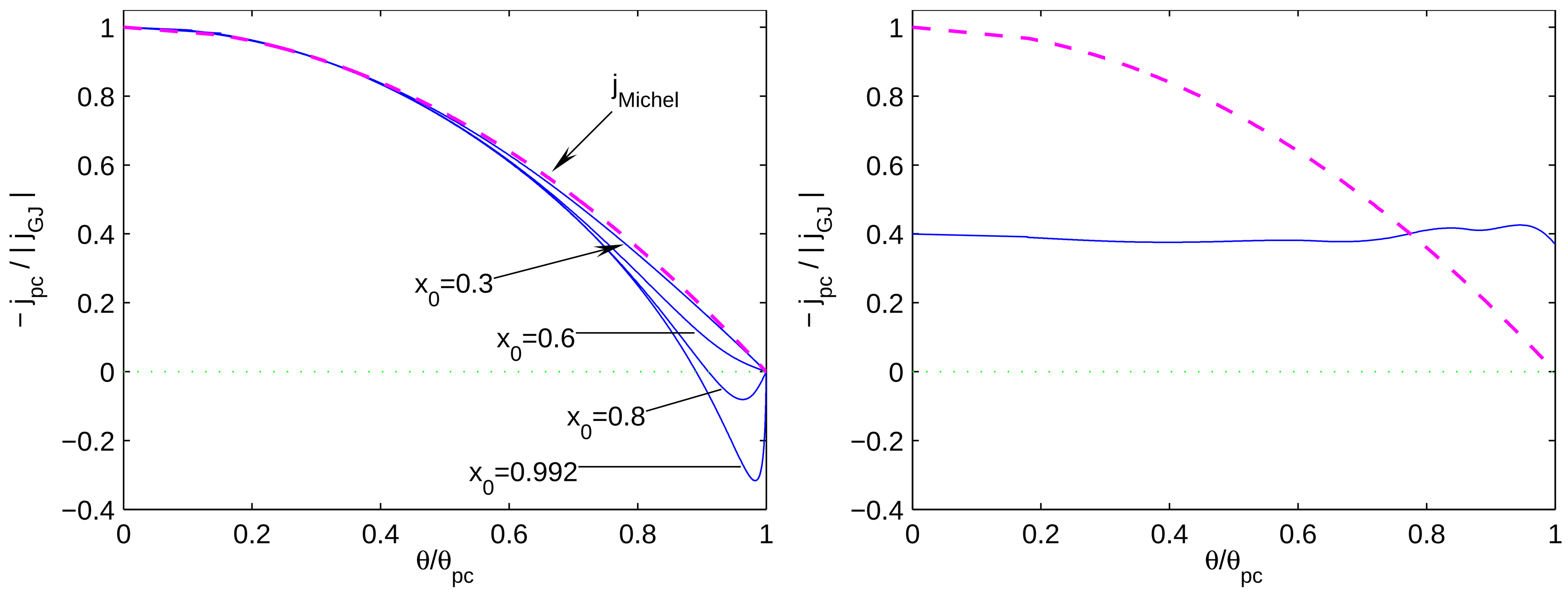

FIGURE 4: *Poloidal current density in the polar cap of pulsar normalized to the Goldreich-Julian current density:* ***left*** *– for $\beta\equiv1$ and different values of x_0;* ***right*** *– for variable β producing nearly constant current density. Michel current density is shown by the dashed line*

Constant $\beta\equiv1$

- Each solution has been checked for applicability of the force-free condition $E<B$. In none of them this condition is violated.
- The total energy of the electromagnetic field in the magnetosphere $\Xi=\int_{Vol}(B^2+E^2)/(8\pi)\,dV$ decreases with increasing of x_0, see Fig. 3 (right).
- Energy losses of the pulsar increase with decreasing of x_0 and are given by (see Fig. 3 (left))

$$W=0.94\,x_0^{-2.065}\times\frac{\mu^2\Omega^4}{c^3}, \quad (7)$$

- In configurations with $x_0>0.6$ there is a *volume* return current flowing along the open magnetic field lines, which makes however only a small part of the whole return current. The current density (in units of $j_{\rm GJ}$) close to the polar cap boundary increases with increasing of x_0. The current density does not exceed the Goldreich-Julian current density $j_{\rm GJ}\equiv\rho_{\rm GJ}c$ and at most field lines it is less than $j_{\rm GJ}$

The problem: current density distribution for $\beta\equiv1$ is not compatible with the Space Charge Limited Flow regime of the stationary polar cap cascades. Current density distribution with return *volume* current is not compatible with any cascade model.

Variable β – magnetosphere with differential rotation in the open field line zone

- Current density can deviate from distributions shown in Fig. 4 (left) and could be made even nearly constant over the polar cap of pulsar. The latter would require adjusting of the plasma angular velocity, i.e. large values of the non-corotational electric potential V. Moreover, in this case the current density j is always less than $j_{\rm GJ}$, see Fig. 4

Conclusions

- The magnetosphere of pulsar should evolve with time, i.e the relative size (in $R_{\rm LC}$) of the closed field line zone should change. This will result in pulsar breaking index different from 3.
- The magnetosphere could rotate differentially – additional degree of freedom for adjusting of the current density required by the global structure of the magnetosphere to the current density produced by the polar cap cascades.
- Electromagnetic cascades in the polar cap of pulsar should be non-stationary.