The MHD diffusion Equation at a 1-D current sheet

Consider a simple current sheet across which the magnetic field reverses so that: $\mathbf{B} = B(x,t)\hat{\mathbf{y}}$.

- Initially, we assume an infinitely thin current sheet, consistent with a step function in the magnetic field: $B(x,0) = \begin{cases} +B_0 & x > 0 \\ -B_0 & x < 0 \end{cases}$

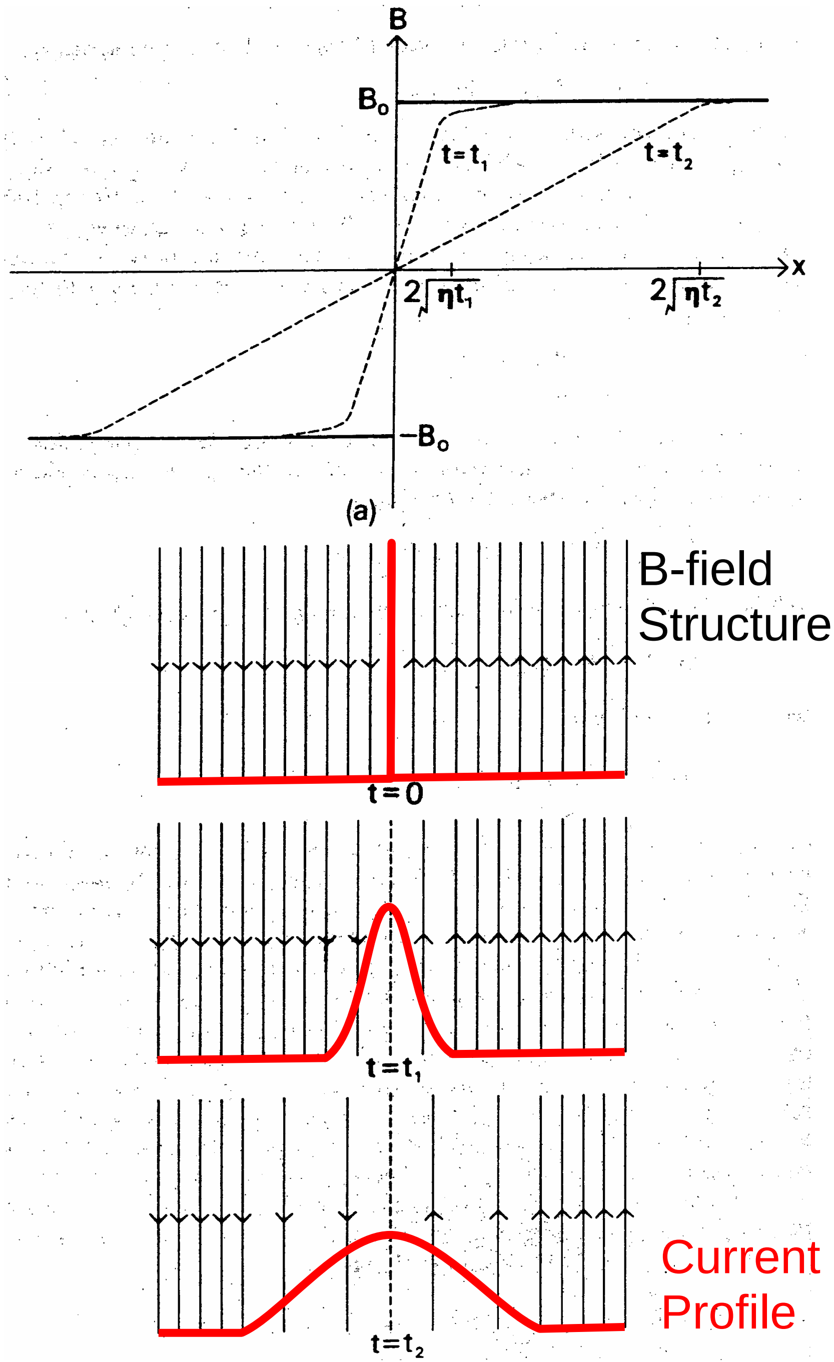
- The diffusion equation in 1-D becomes $\frac{\partial B}{\partial t} = \frac{\eta}{\mu_o}\frac{\partial^2 B}{\partial x^2}$.

- There are standard solutions to such equations of the form: $B(x,t) = B_0 erf\left[\left(\frac{\mu_0}{4\eta t}\right)^{1/2} x\right]$.

(erf (ξ) is the *error function*, defined by $erf(\xi) = \frac{2}{\sqrt{\pi}}\int_0^{\xi} e^{-u^2}du$, which has standard solutions that can be looked up in tables of integrals, etc.)

- The solution is such that the magnetic field diffuses and annihilates, as illustrated in the figure to the left.

- Notice the current sheet also broadens, and the current density, $j_z = \frac{1}{\mu_o}\frac{\partial B}{\partial x}$, weakens but the total current J in the sheet (area under the current curve) remains constant, i.e. $J = \int_{-\infty}^{\infty} j_z dx = \frac{2B_0}{\mu_0} = Const.$


B
B_o
$t=t_1$
$t=t_2$
x
$2\sqrt{\eta t_1}$
$2\sqrt{\eta t_2}$
$-B_o$
(a)
B-field
Structure
t=0
$t=t_1$
$t=t_2$
Current
Profile