Single Particle Picture of the Magnetopause Current Layer (NB Examinable)
(discussion relates to figure labelled similarly)

i) Solar wind protons and electrons of number density n_{SW} arrive at magnetopause boundary with the solar wind velocity $\mathbf{u} = -u_{SW}\hat{\mathbf{x}}$. (NB no gyromotion upstream as we have ignored any solar wind magnetic field).

ii) At boundary, they encounter magnetospheric field $\mathbf{B} = B\,\hat{\mathbf{z}} = 2\,B_E(r)\,\hat{\mathbf{z}}$ tangential to boundary.

iii) Each particle performs a half gyration in x-y plane, before returning to the solar wind side with $\mathbf{u} = +u_{SW}\hat{\mathbf{x}}$. Recall that ion gyroradius $r_{Lp} >> r_{Le}$, the electron gyroradius ($r_L = u_{SW}\, m / qB \propto m$).

iv) Ions gyrate in the opposite sense to electrons, so paired ions and electrons separate at the magnetopause, giving a current in the y-direction.

v) Consider protons crossing a given plane of constant $y = y_o$ during their gyration. These particles must have entered the magnetopause layer within $2\, r_{Lp}$ of this plane (else they couldn't cross it during their gyration).

vi) So the number of protons crossing the $y = y_o$ plane per height element dz is the incoming flux of particles crossing an area $2\, r_{Lp}\, dz$, i.e.

$$N_p = n_{SW}\; u_{SW}\; 2\, r_{Lp}\; dz$$

vii) Similarly, for electrons, $N_e = n_{SW}\; u_{SW}\; 2\, r_{Le}\; dz$.

viii) So the current through the plane

$$\begin{aligned} I &= q_p N_p - q_e N_e \\ &= q\,[n_{SW}\; u_{SW}\; 2\, r_{Lp} - n_{SW}\; u_{SW}\; 2\, r_{Le}]\, dz \\ &= q\, n_{SW}\; u_{SW}\; 2\,[r_{Lp} - r_{Le}]\, dz \\ &= q\, n_{SW}\; u_{SW}\; 2\, r_{Lp}\; dz \end{aligned}$$

(as $q_p = -q_e = q\ (=e)$, $r_{Lp} >> r_{Le}$)

Hence the magnetopause current is *proton-dominated* (more protons cross a given plane).

ix) Substituting for r_{Lp},

$$I = \frac{2 n_{SW} m_p u_{SW}^2}{B} dz$$

$$or \quad n_{SW} m_p u_{SW}^2 = \frac{B\, I}{2\, dz}$$

x) Recall Stoke's Theorem and Ampere's Law:

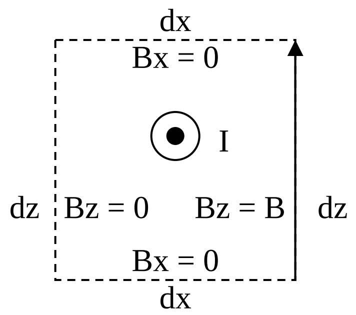
$$\int_C \mathbf{B.dl} = \oint_S (\nabla \times \mathbf{B}).\mathbf{dS} \quad \text{and} \quad \nabla \times \mathbf{B} = \mu_o\, \mathbf{j}$$

$$\Rightarrow \quad \int_C \mathbf{B.dl} = \oint_S \mu_o\, \mathbf{j.dS}$$

and consider a square loop of height dz and width dx in the magnetopause current layer:

$$\int_C \mathbf{B.dl} = B\, dz$$

$$\oint_S \mu_o\, \mathbf{j.dS} = \mu_o\, I$$



$$\text{So} \quad B\, dz = \mu_o \frac{2 n_{SW} m_p u_{SW}^2}{B} dz$$

$$\text{or} \quad m_p n_{SW} u_{SW}^2 = \frac{B^2}{2\, \mu_o}$$

In other words the Ram pressure of the solar wind balances the magnetic pressure in the magnetosphere, the same result as obtained considering MHD pressure balances (note again that B = 2 B_E(r) for the Earths dipole case).

N.B. The thickness of the current sheet dx ~ r_{Lp} ~ a few hundred km. (This is small compared to the height/width scale lengths which are many R_E.) *So technically MHD may not be valid here.* However, as with shocks, we again see that this is a case where MHD provides an adequate description of the plasmas around the boundary without having to consider the microphysics operating within the thin layer.