Lecture 13: Gravitational Waves

1 Introduction

This lecture will discuss gravitational waves. We will see what they are, how they are made, how they can be detected indirectly, how they can be detected directly, and what are the first results from gravitational wave astronomy.

2 What are gravitational waves?

Gravitational waves are waves in the gravitational field. To imagine how these might come about, think first of the tides on the Earth. The differential gravitational forces due to the difference in distance between the moon and the near and far parts of the Earth induces the tides. Now imagine if the Sun was a binary star rotating rapidly, and you were sampling the gravitational field on the Earth with a test particle. We can imagine that the test particle might oscillate because of the changing gravitational field from the rotating binary star. Thus we can see that oscillating or rotating bodies can produce gravity waves, somewhat analogous to the way that an oscillating charge produces electromagnetic waves.

However, gravity waves are a little different to electromagnetic waves. Firstly, mass behaves differently to charge: mass is involved in gravity in a similar way that charge is involved in electromagnetism, but mass is also involved in inertia and momentum. From conservation of momentum, whenever mass is accelerated in one direction, other masses experience equal and opposite change of momentum in the other direction. Unlike electromagnetic radiation, gravitational radiation does not depend on the dipole moment, so it doesn’t just move things up and down.

3 Why are gravitational waves important?

- gravitational waves are predicted by Einstein’s theory of general relativity; they are an important, testable prediction of the theory.
- they have important observational consequences: they can be a significant energy loss mechanism, and they can cause compact orbits to decay.
- Perhaps most importantly, they are a very different means of astronomical observation: they could be even more revolutionary than X-ray astronomy or radio astronomy!

4 So, what do gravity waves do?

Gravity waves cause small fractional distortions in the distance between two points; this is called the ‘gravitational strain’. Gravity waves have a quadrupole effect, so that when points move further away in one dimension, points move closer together in the other dimension, as shown in figure 1.

Just like electromagnetic waves, gravitational radiation is able to cause acceleration in external systems, and so must carry energy. If the separation between two orbiting bodies is \( l \), then the emitted power in gravitational waves can be written:

\[
P_{GW} = \frac{c^5}{G} \left( \frac{GM}{c^2} \right)^5
\]

where \( M \) is the mass of the system, \( c \) is the speed of light and \( G \) is the gravitational constant. By writing the equation in this way, we can see that there is a maximum power that can be emitted by any system. \( GM/c^2 \) is
half the Schwarzschild radius; the dimensions of the orbiting system $l$ cannot be less than half the Schwarzschild radius, so the **maximum** power that can be emitted is $c^5 / G$, $\sim 4 \times 10^{52}$ W, regardless of the mass of the system.

For the Earth-Sun orbital system, the amount of gravitational radiation expected is 200 W; we should keep on orbiting the Sun for another $10^{23}$ years before the Earth’s orbit decays because of gravitational radiation (not much to worry about...) However, when the dimensions of the orbit approach the Schwarzschild radius then gravitational radiation can be a very serious energy loss mechanism.

From equation 1 we can think of some obvious expected strong sources of gravitational waves.

- very compact binaries will lose energy rapidly by gravitational radiation.
- if short $\gamma$-ray bursts are produced by the merging of two neutron stars or a black hole with a neutron star, there should be a strong gravitational wave signal just before the burst as the orbit decays rapidly.

5 **Indirect detection**

Gravitational waves remove energy from an orbiting system. If we can find a system with well defined orbital parameters, and with a sufficiently compact orbit that it should be a significant source of gravitational radiation, then we can predict how fast the orbit should be decaying due to gravitational radiation, and compare this with the rate of orbital changes measured in the system.

A binary system of two neutron stars, one of which is a pulsar, was discovered in 1975 by Joseph Taylor and Russel Hulse. The orbital period is 7.8 hours and the pulsar pulse period is 0.059s. The timing of the pulses combined with general relativity theory, allows the orbital parameters to be measured very accurately. The observed rate of change in the orbital period (i.e. the amount that the period (in seconds) changes by every second) is $-2.4 \pm 0.4 \times 10^{-12}$, consistent with the predicted period change due to gravitational radiation. This work won Taylor and Hulse the 1993 Nobel prize for physics!

The binary pulsar is an excellent system to test the theory of gravitational radiation, but it is not the brightest predicted source of gravitational waves in the Galaxy. In the last 15 years a new class of binary star, called ‘ultracompact binaries’ have been discovered using X-ray surveys. There are now at least 3 such systems known; they are double-degenerate binaries with orbital periods less than 10 minutes. These ought to be very bright persistent sources of gravitational radiation.

6 **Direct detection**

As I have already said, the principal observational effect of gravitational waves is a small fractional distortion in the displacement between two points (i.e. change in distance divided by the distance), called the gravitational strain. Direct detection of gravitational waves involves detecting these small distortions. For reference, the strain expected from a maximally emitting source at a distance of 1 kpc, at a frequency of 1 kHz, is only about $10^{-14}$. Realistic expectations for the strain from a supernova forming a neutron star 1 Mpc away is more like $10^{-20}$. This is an absurdly small quantity to measure, akin to adding an atomic nucleus to a 1 AU baseline.

There are two types of detector that have been widely proposed for detecting gravitational radiation: resonant bars and laser interferometers.

7 **Resonant bars**

These were pioneered by J. Weber. Gravitational strain will make bars resonate, rather like ringing a bell. In 1969 Weber published his claimed detection of gravitational radiation, typically 3 bursts per day, using aluminium
cylinders about 2m long and with resonant frequency of about 1.6 kHz. However, Levine and Garwin failed to detect any such gravity waves using similar apparatus and reported this in 1973. The resonances detected by Weber were not due to gravitational waves, and resonant bars have not successfully detected Gravitational waves to date.

8 Laser interferometers

New gravitational wave experiments are based on laser interferometers. Laser light is transmitted along two perpendicular arms, reflected at each end, and recombined at the centre to produce interference fringes. Thus the path lengths along the arms can be measured to a fraction of the wavelength of the laser light. Changes in the path lengths as a gravitational wave passes over the interferometer will produce a change in the interference fringes. This is a technically challenging experiment. There have been several significant interferometers built, including the German/UK ‘Geo 600’ interferometer in Hanover. This has two 600m long arms, and the light path through each arm is kept in vacuum over the whole length. It was important for the development of the technology of gravitational wave interferometers, but did not detect any gravity waves.

The most advanced interferometer experiment is called the Laser Interferometer Gravitational-wave Observatory (LIGO), and consists of two interferometers, one at Livingston and one at Hanford in the USA. Two interferometers allow gravity waves to be distinguished from terrestrial interference such as local seismic disturbances, which will only register on one interferometer. They also provide some information on the location of gravitational wave sources in the sky from the light (and therefore gravitational wave) travel time between the two interferometers. Each interferometer has two 4 km-long arms, with the whole of the light path in vacuum. The next best interferometer, with 3km long arms, is the European VIRGO Gravitational wave observatory located near Pisa in Italy.

9 Detection of gravitational waves with LIGO

On 11th February 2016 the international LIGO consortium announced the direct detection of gravitational waves. The announcement followed decades of development of the LIGO interferometers (these days they are called “Advanced LIGO” because of the substantial improvements in sensitivity compared to the earlier technology). The first gravitational waves to be directly detected were a brief event which happened on the 14th September 2015, known as GW150914, which, it was concluded, came from the merging of two black holes of masses 36 and 29 solar masses. The characteristic ‘chirp’ that comes from the merging of compact bodies, in which both frequency and strain increase towards the moment of merging, was detected almost simultaneously at the two LIGO interferometers, lasted about 0.2 seconds and had a maximum strain of $10^{-21}$. Soon afterwards, the detection of GW151226 was announced, which was the merger of black holes of masses 14 and 8 solar masses. The two detections have important astrophysical consequences. These are remarkably massive black holes to find in binary star systems. They are likely to have originated in dense stellar clusters where binary black holes can be created dynamically as opposed to coming from the isolated evolution of a binary system of two massive stars. The progenitors of the black holes in GW150914 probably came from massive, but metal-poor stars; even (initially) very massive stars are thought to lose too much of their mass through stellar winds before the end of their lives to leave black holes this massive if their metallicities were similar to that of the Sun.

LIGO detected three more black hole mergers, before the first neutron-star - neutron star merger was detected in August 2017: GW170817. By this time the European VIRGO interferometer was also working, and the addition of the VIRGO data allowed much better constraints on the position in the sky of the event than the LIGO data alone. Within 2 seconds of the conclusion of the gravitational wave signal from GW170817 a short gamma-ray burst was detected with the NASA Fermi satellite, from a consistent location in the sky to the gravitational wave signal. Unfortunately, Swift was on the wrong side of the Earth to detect this burst at the time, so the sky position was rather poorly known. After about half a day of searching by myriad ground and spaced-based observatories, the optical counterpart was identified in the nearby galaxy NGC4993. Observations with Swift UVOT showed it to be an ultraviolet-bright source, consistent in properties with those expected from a ‘kilonova’, a weak supernova that is predicted to follow a neutron star merger. In a single day, the neutron star merger origin for short gamma ray bursts was proven, and the reality of gravitational wave signals proved beyond any doubt by the detection of the same event in electromagnetic radiation.

10 Space interferometers
The Laser Interferometer Space Antenna (LISA) has great potential as an astrophysical gravitational wave observatory, working at lower frequencies than LIGO. LISA is a proposal to launch a 3-spacecraft laser interferometer into space. This way, much longer baselines of 5 million km can be achieved. A whole host of astrophysical systems should be detected (and distinguished) by LISA, including many binaries within our own Galaxy. However the technical challenge is extraordinary because the 3 spacecraft will have to station-keep with a precision equal to a fraction of the wavelength of light, at spacecraft separations of 5 million km. The European Space Agency has LISA in its programme as a large mission to be launched in 2034. A technology-proving mission called LISA-Pathfinder recently demonstrated that the technologies required for LISA are feasible.

11 The future

The direct detection of gravitational waves could mark the beginning of a whole new type of astronomy. Potentially we may discover other new types of astrophysical system that we have never witnessed with electromagnetic radiation (already we have seen the binary black holes). Gravitational waves might one day allow us to probe directly the early Universe before recombination, because gravitational waves are unimpeded by electron scattering, which renders the Universe opaque to electromagnetic observation earlier than this time.

12 Key points

- Gravitational radiation is a phenomenon predicted by general relativity.
- It is (sort of) the gravitational equivalent of electromagnetic radiation.
- Gravitational waves cause small fractional distortions to the displacement between points in space.
- Gravitational radiation is an important energy loss mechanism in very close binaries.
- Gravitational waves have been indirectly detected using the binary pulsar.
- Laser interferometers have directly detected gravitational waves from merging binary black holes and merging neutron stars.
- This is a truly revolutionary new type of astronomy!