Multi-Scale 3-D Surface Description: Open and Closed Surfaces

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Abstract

A novel technique for multi-scale smoothing of a free-form 3-D surface is presented. Complete triangulated models of 3-D objects are constructed automatically and using a local parametrization technique, are then smoothed using a 2-D Gaussian filter. Our method for local parametrization makes use of semigeodesic coordinates as a natural and efficient way of sampling the local surface shape. The smoothing eliminates the surface noise together with high curvature regions such as sharp edges, therefore, sharp corners become rounded as the object is smoothed iteratively. Our technique for free-form 3-D multi-scale surface smoothing is independent of the underlying triangulation. It is also argued that the proposed technique is preferrable to *volumetric smoothing* or *level set methods* since it is applicable to incomplete surface data which occurs during occlusion. Our technique was applied to closed as well as open 3-D surfaces and the results are presented here.

1 Introduction

This paper introduces a new technique for multi-scale shape description of free-form 3-D surfaces represented by polygonal or triangular meshes. Complete 3-D models of test objects have been used in our experiments. Such models can be constructed through automatic fusion of range images of the object obtained from different viewpoints [3].

Multi-scale descriptions have become very common in computer vision since they offer added robustness with respect to noise and object detail as well as provide for more efficient processing. The multi-scale technique proposed here can be considered a generalization of earlier multi-scale representation theories proposed for 2-D contours [7, 8] and space curves [5]. However, the theoretical issues are significantly more challenging when working on free-form 3D surfaces.

In our approach, diffusion of the surface is achieved through convolutions of local parametrizations of the surface with a 2-D Gaussian filter. *Semigeodesic coordinates* [2] are utilized as a natural and efficient way of locally parametrizing surface shape. The most important advantage of our method is that unlike other diffusion techniques such as volumetric diffusion [4] or level set methods [11], it has *local support* and is therefore applicable to partial data corresponding to surface-segments. This property makes it suitable for object recognition applications in presence of occlusions.

The organization of this paper is as follows. Section 2 gives a brief overview of previous work on 3-D object representations including the disadvantage(s) of each method. Section 3 describes the relevant theory from differential geometry and explains how a multi-scale shape description can be computed for a free-form 3-D surface. Section 4 covers implementation issues encountered when adapting semigeodesic coordinates coordinates to 3-D triangular meshes. Section 5 presents diffusion results and discussion. Section 6 contains the concluding remarks.

2 Literature Survey

Polyhedral approximations [1] fit a polyhedral object with vertices and relatively large flat faces to a 3-D object. Their disadvantage is that the choice of vertices can be quite arbitrary which renders the representation not robust. Smooth 3-D splines [13] can also be fitted to 3-D objects. Their shortcomings are that the choice of knot points is again arbitrary and that the spline parameters are not invariant. *Generalized cones* or *cylinders* [12] as well as *geons* [9] approximate a 3-D object using globally parametrized mathematical models, but they are not applicable to detailed free-form objects. *Multi-view* representations [10] are based on a large number of views of a 3-D object obtained from different viewpoints, but difficulties can arise when a non-standard view is encountered. In *volumetric diffusion* [4] or level set methods [11], an object is treated as a filled area or volume. The object is then blurred by subjecting it to the diffusion equation. The boundary of each blurred object can then be defined by applying the Laplacian operator to the smoothed area or volume. The major shortcoming of these approaches is lack of local support. In other words, the entire object data must be available. This problem makes them unsuitable for object recognition in presence of occlusion. A form of 3-D surface smoothing has been carried out in [14] but this method has drawbacks since it is based on weighted averaging using neighboring vertices and is therefore dependent on the underlying triangulation.

3 Semigeodesic Parametrization on a 3-D Surface

A crucial property of 2-D contours and space curves (or 3-D contours) is that they can be parametrized globally using the arclength parameter. However, free-form 3-D surfaces are more complex. As a result, no global coordinate system exists on a free-form 3-D surface which could yield a natural parametrization of that surface. Indeed, studies of local properties of 3-D surfaces are carried out in differential geometry using local coordinate systems called *curvilinear coordinates* or *Gaussian coordinates* [2]. Each system of curvilinear coordinates is introduced on a patch of a regular surface referred to as a *simple sheet*. A simple sheet of a surface is obtained from a rectangle by stretching, squeezing, and bending but without tearing or gluing together. Given a parametric representation $\mathbf{r} = \mathbf{r}(u,v)$ on a local patch, the values of the parameters u and v determine the position of each point on that patch.

3.1 Geodesic Lines

Before the semigeodesic coordinates can be described, it is necessary to define geodesic lines on a regular 3-D surface. The following definitions are useful [2]:

Definition. A geodesic line or a geodesic of a surface is a curve whose *geodesic curvature* is zero at every point. Geodesic curvature is the magnitude of the *vector of geodesic curvature.*

Definition. The vector of geodesic curvature of a curve C lying on a surface $\mathcal{S}$ at a point P on C is obtained by projecting the *curvature vector* of C at P on the tangent plane to $\mathcal{S}$ at P.

Definition. The curvature vector of a curve C at point P is of the same direction as the principal normal vector at P and of length equal to the curvature of the curve at P.

Definition. The principal normal vector of a curve C at point P is perpendicular to C at P and lies in the *osculating plane* at P. The plane with the highest possible order of contact with the curve C at point P is called the osculating plane at P.

The following crucial property of geodesic lines is actually utilized to construct geodesics on 3-D triangular meshes:

Minimal property of geodesics: An arc of a geodesic line C passing through a point P and lying entirely in a sufficiently small neighborhood of a point P of a surface $\mathcal{S}$ of class C_2 is the shortest join of P with any other point of C by a curve lying in the neighborhood.

3.2 Semigeodesic Coordinates

Semigeodesic coordinates can be constructed in the following way at a point P on a surface $\mathcal{S}$ of class C_2:

- Choose a geodesic line C through point P in an arbitrary direction.
- Denote by v the arclength parameter on C, such that P corresponds to the value $v = 0$.
- Take further through every point of C the geodesic line L perpendicular to C at the corresponding point.
- Denote by u the arclength parameter on L.

The two parameters u and v determine the position of each point in the domain swept out by these geodesic lines. It can be shown that in a sufficiently small neighborhood of the point P, semigeodesic coordinates can always serve as curvilinear coordinates in a regular parametric representation of $\mathcal{S}$ [2]. The orthogonal cartesian coordinates in the plane are a special case of semigeodesic coordinates on a flat surface.

3.3 Gaussian Smoothing of a 3-D surface

The procedures outlined above can be followed to construct semigeodesic coordinates at every point of a 3-D surface $\mathcal{S}$. In case of semigeodesic coordinates, local parametrization yields at each point P:

$$\mathbf{r}(u,v) = (x(u,v),\ y(u,v),\ z(u,v)).$$

The new location of point P is given by:

$$\mathbf{R}(u,v,\sigma) = (\mathcal{X}(u,v,\sigma),\ \mathcal{Y}(u,v,\sigma),\ \mathcal{Z}(u,v,\sigma))$$

where

$$\mathcal{X}(u,v,\sigma) = x(u,v)\ \otimes\ G(u,v,\sigma)$$
$$\mathcal{Y}(u,v,\sigma) = y(u,v)\ \otimes\ G(u,v,\sigma)$$
$$\mathcal{Z}(u,v,\sigma) = z(u,v)\ \otimes\ G(u,v,\sigma)$$

$$G(u,v,\sigma) = \frac{1}{2\pi\sigma^2}\ e^{-\frac{(u^2+v^2)}{2\sigma^2}}$$

and $\otimes$ denotes convolution.

This process is repeated at each point of $\mathcal{S}$ and the new point positions after filtering define the smoothed surface. Since the coordinates constructed are valid locally, the Gaussian filters always have $\sigma = 1$.

3.4 Multi-Scale Description of a 3-D surface via diffusion

In order to achieve multi-scale descriptions of a 3-D surface $\mathcal{S}$, it is smoothed according to the process described in section 3.3. The smoothed surface is then considered as the input to the next stage of smoothing. This procedure is then iterated many times to obtain multi-scale descriptions of $\mathcal{S}$. This process is equivalent to diffusion smoothing

$$\frac{\partial \mathcal{S}}{\partial t} = H\,\mathbf{n}$$

since the Gaussian function satisfies the heat equation. In the equation above, t is time, H is mean curvature, and $\mathbf{n}$ is the surface normal vector. t can be regarded as the number of iterations.

4 Implementation on a 3-D Triangular Mesh

The theory explained in the previous section must be adapted to a 3-D triangular mesh. Semigeodesic coordinates involve construction of geodesic lines. Clearly the segment of a geodesic that lies on any given triangle is a straight line. Two situations must be considered:

- Extension of a geodesic when it intersects an edge
- Extension of a geodesic when it intersects a vertex

Figure 1 illustrates the first case and figure 2 illustrates the second case.

Theorem 1 addresses the first situation.

Theorem 1. Suppose a geodesic intersects an edge e shared by triangles T_1 and T_2. The extension of this geodesic beyond e is obtained by rotating T_2 about e so that it becomes co-planar with T_1, extending the geodesic in a straight line on T_2, and rotating T_2 about e back to its original position.

Theorem 2 addresses the second situation.

Theorem 2. Suppose a geodesic arrives at a vertex V of the mesh. Define the normal vector $\mathbf{n}$ at V as the average of the surface normals of all the triangles incident on V weighted by the incident angle. Let Q be the plane formed by the geodesic incident on V and $\mathbf{n}$. The extension of this geodesic beyond V is found by intersecting Q with the mesh.

Proofs of these theorems are given in [6].

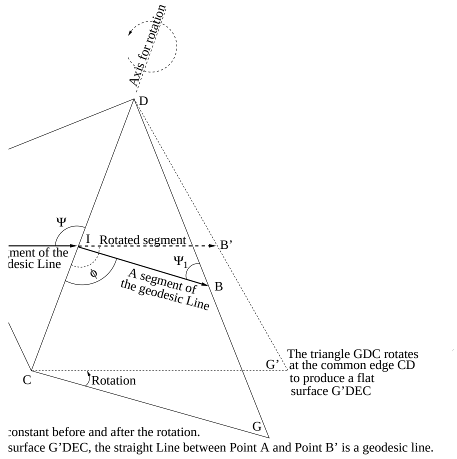


Figure 1: A geodesic intersects a triangle edge

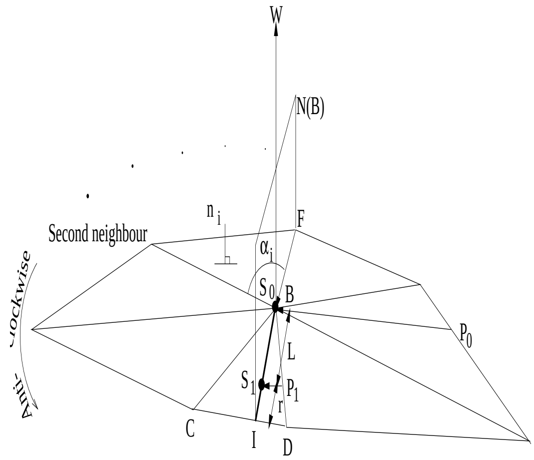
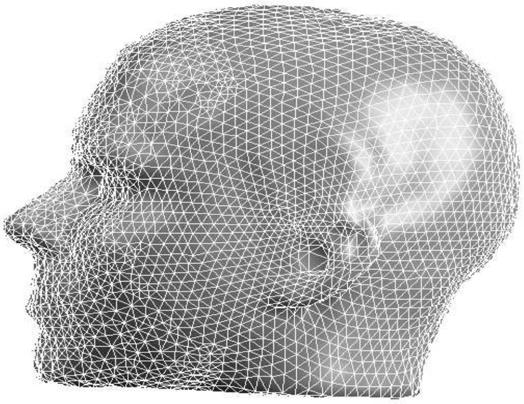


Figure 2: A geodesic intersects a triangle vertex

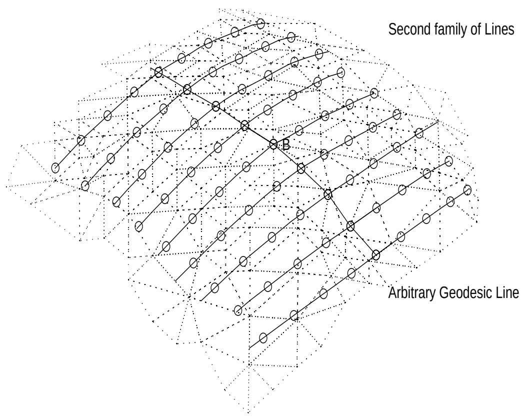
4.1 Implementation of semigeodesic coordinates

Semigeodesic coordinates are constructed at each vertex of the mesh which becomes the local origin. The following procedure is employed:

1. Construct a geodesic from the origin in an arbitrary direction such as the direction of one of the incident edges.
2. Construct the other half of that geodesic by extending it through the origin in the reverse direction using the procedure outlined in theorem 2.
3. Parametrize that geodesic by the arclength parameter at regular intervals to obtain a sequence of sample points.
4. At each sample point on the first geodesic, con-



(a) A 3-D triangular mesh



(b) Semigeodesic coordinates

Figure 3: A triangular mesh and semigeodesic coordinates for an area at the back of the head at about eye level

struct a perpendicular geodesic and extend it in both directions.

5. Parametrize each of the geodesics constructed in the previous step by the arclength parameter at regular intervals.

Figure 3 shows a triangular mesh and an example of semigeodesic coordinates.

4.2 Semigeodesic coordinates on open surfaces

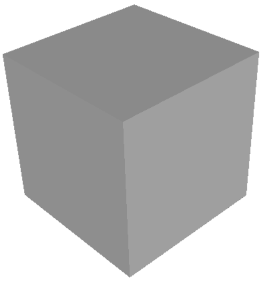
Quite often, due to occlusion or complex object shape, it is not possible to construct complete and closed surfaces. As a result, the algorithm described above should be modified to make it also applicable to open surfaces. The algorithm for smoothing an open surface is defined in the following way:

1. Grid construction and smoothing at internal vertices is carried out as on closed surfaces. Any geodesic line that reaches the boundary will stop. The last sample point at or near the boundary will be duplicated until the grid is filled. Likewise, if some geodesic lines can not be constructed, the last geodesic line near the boundary will be duplicated until the grid is filled.
2. If the vertex V of triangle T resides on the boundary, measure the angle α between the two edges of T that are incident on V. Choose the first geodesic line as the bisector of α. Only half of the first geodesic line is constructed because the other half falls outside the surface boundary.
3. At the same vertex, construct another geodesic line perpendicular to the first one.
4. One of those geodesic lines might soon intersect the boundary, so compare the lengths of those lines and choose the longer one. This allows the maximum size grid to be constructed.
5. Construct the second family of geodesic lines as perpendicular to the longer geodesic line determined above.
6. As before, any geodesic line that reaches the boundary will stop, and the last sample point at or near the boundary will be duplicated until the grid is filled.

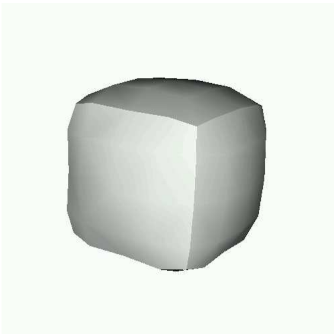
5 Results and Discussion

The smoothing routines were implemented entirely in C++. Complete triangulated models of 3-D objects used for our experiments are constructed at our center [3]. Each iteration of smoothing on a surface with 1000 vertices took about 0.5 second of CPU time on an UltraSparc 170E. In order to experiment with our techniques, both simple and complex 3-D objects with different numbers of triangles were used. The first test object was a simple cube with 98 vertices. The smoothing results using semigeodesic coordinates are shown in Figure 4. The original cube becomes rounded iteratively and finally changes to a sphere after five iterations.

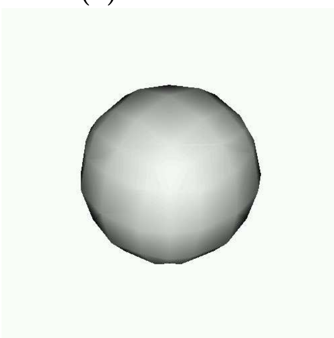
The second test object was a foot with 500 vertices. The smoothing results are shown in figure 5. The foot also becomes rounded iteratively and evolves into an ellipsoidal shape after 100 iterations. We then examined our technique with more complex 3-D objects. Figure 6 shows the third test object which was a telephone handset with 1000 vertices. Notice that the surface noise is eliminated iteratively with the object becoming smoother gradually and the handset almost disappears



(a) Original

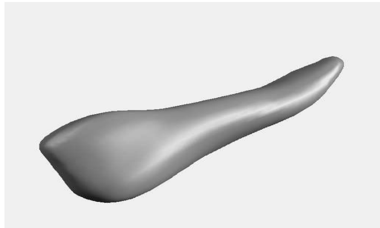


(b) 1 iteration

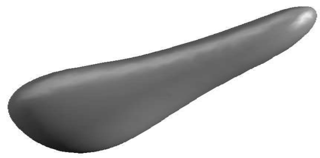


(c) 5 iterations

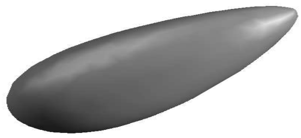
Figure 4: Smoothing of the Cube



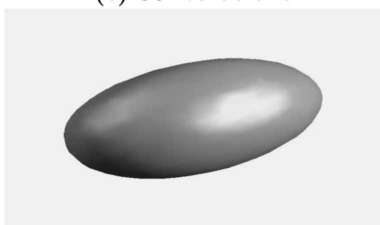
(a) Original



(b) 10 iterations



(c) 30 iterations



(d) 100 iterations

Figure 5: Diffusion of the Foot

after 15 iterations. The fourth test object was a chair with 1000 vertices as shown in figure 7 where again the legs of the chair disappear as was seen for the telephone handset. Therefore, from these results one can conclude that the proposed technique is effective in eliminating the surface noise as well as removing surface detail and it is also independent of the underlying triangulation. The result is gradual simplification of object shape. Animation of surface diffusion can be observed at:

http://www.ee.surrey.ac.uk/Research/VSSP/demos/css3d/

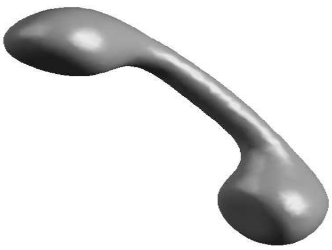
Our smoothing technique was also applied to a number of open/incomplete surfaces. Figure 8 shows the smoothing results obtained on a part of the foot object shown in figure 5. Figure 9 shows the results obtained on a part of the telephone handset shown in figure 6. This object also has a triangle removed in order to generate an internal hole. Figure 10 shows the smoothing results obtained on the lower part of the chair object shown in figure 7. Figure 11 shows smoothing results obtained on a partial rabbit.

6 Conclusions

A novel technique for multi-scale smoothing of a free-form triangulated 3-D surface was presented. This was achieved by convolving local parametrizations of the surface with 2-D Gaussian filters iteratively. Our method for local parametrization made use of semigeodesic coordinates as natural and efficient ways of sampling the local surface shape. The smoothing eliminated the surface

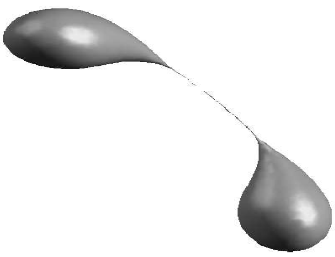
(a) Original

(b) 2 iterations



(c) 6 iterations

(d) 12 iterations



(e) 15 iterations

Figure 6: Diffusion of the telephone handset

(a) Original

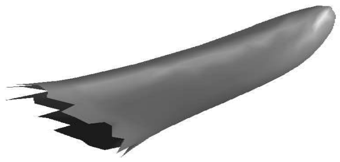
(b) 2 iterations

(c) 3 iterations

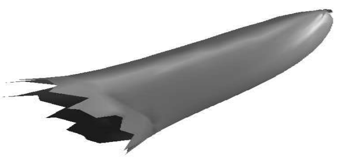
(d) 4 iterations

Figure 7: Smoothing of the Chair

(a) Original



(b) 2 iterations



(c) 5 iterations

Figure 8: Diffusion of the partial Foot

(a) Original

(b) 3 iterations

(c) 5 iterations

Figure 9: Diffusion of the partial telephone handset

noise and small surface detail gradually, and resulted in gradual simplification of object shape. The method was independent of the underlying triangulation. Our approach is preferrable to *volumetric smoothing* or *level set methods* since it is applicable to incomplete surface data which occurs during occlusion.

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(a) Original

(b) 2 iterations

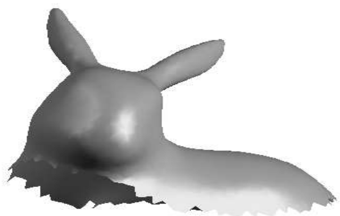
(c) 3 iterations

(d) 4 iterations

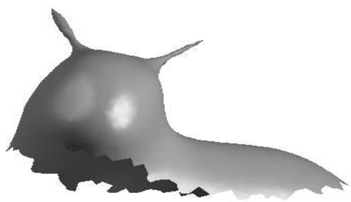
Figure 10: Smoothing of the partial Chair

(a) Original

(b) 2 iterations



(c) 5 iterations



(d) 9 iterations

Figure 11: Smoothing of the rabbit head