Recovery of Curvature and Torsion Features from Free-Form 3-D Meshes at Multiple Scales

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Abstract

A novel technique for multi-scale curvature computation on a smoothed 3-D surface is presented. This is achieved by convolving local parametrisations of the surface with 2-D Gaussian filters iteratively. In our technique, semigeodesic coordinates are constructed at each vertex of the mesh which becomes the local origin. A geodesic from the origin is first constructed in an arbitrary direction such as the direction of one of the incident edges. The smoothing eliminates surface noise and small surface detail gradually, and results in gradual simplification of the object shape. The surface Gaussian and mean curvature values are estimated accurately at multiple scales together with curvature zero-crossing contours. The curvature values are then mapped to colours/grey-scales and displayed directly on the surface. Furthermore, local maxima of Gaussian and mean curvatures as well as the torsion maxima of the zero-crossing contours of Gaussian and mean curvatures were also located and displayed on the surface. These features can be utilised by later processes for robust surface matching and object recognition. Our technique is independent of the underlying triangulation and is also more efficient than volumetric diffusion techniques since 2-D rather than 3-D convolutions are employed. Another advantage is that it is applicable to incomplete surfaces which arise during occlusion or surfaces with holes.

Keywords: Free-form surfaces, Local parametrisation, Semigeodesic coordinates, Multi-scale smoothing, Gaussian and Mean curvatures, Curvature zero-crossing contours, Local curvature maxima, Torsion maxima of zero-crossing contours.

1 Introduction

Curvature estimation is an important task in 3-D object description and recognition. Surface curvature provides a unique view-point invariant description of local surface shape. Differential geometry [1] provides several measures of curvature, which include Gaussian and mean curvatures. Combination of these curvature values enable the local surface type to be categorised.

This paper introduces a new technique for multi-scale curvature computation on a smoothed 3-D surface. Complete triangulated models of 3-D objects are constructed and using a local parametrisation technique, are then smoothed using a 2-D Gaussian filter. The technique considered here is a generalisation of earlier multi-scale representation theories proposed for 2-D contours [5] and space curves [3]. More details of the diffusion technique as well as literature survey appear in [4].

In our approach, diffusion of the surface is achieved through convolutions of local parametrisations of the surface with a 2-D Gaussian filter [4, 8]. *Semigeodesic coordinates* [1] are utilised as a natural and efficient way of locally parametrising surface shape. The most important advantage of our method is that unlike other diffusion techniques such as volumetric diffusion [2] or level set methods [7], it has *local support* and is therefore applicable to partial data corresponding to surface-segments. This property makes it suitable for object recognition applications in presence of occlusions. It is also more efficient than those techniques since 2-D rather than 3-D convolutions are employed.

The paper contains examples showing 3-D objects with their Gaussian and mean curvature values estimated. To visualise these curvature values on the surface, they are then mapped to colours/grey-scales. Colour mapping is a scalar visualisation technique provided in a software package called Visualisation Toolkit (VTK) [6]. Once surface curvatures are estimated, then curvature zero-crossing contours are recovered and displayed on the surface. Finally, local maxima of Gaussian and mean curvatures as well as the maxima of torsion of zero-crossing contours of Gaussian and mean curvatures were also located and displayed on the surface.

The organisation of this paper is as follows. Section 2 describes the relevant theory from differential geometry and explains how a multi-scale shape description can be computed for a free-form 3-D surface. Section 3 covers the computation of Gaussian and mean curvatures as well as their zero-crossing contours and maxima. Section 4 presents results and discussion. Section 5 contains the concluding remarks.

2 Semigeodesic Coordinates

Free-form 3-D surfaces are complex hence, no global coordinate system exists on these surfaces which could yield a natural parametrisation of that surface. Studies of local properties of 3-D surfaces are carried out in differential geometry using local coordinate systems called *curvilinear coordinates* or *Gaussian coordinates* [1]. Each system of curvilinear coordinates is introduced on a patch of a regular surface referred to as a *simple sheet*. A simple sheet of a surface is obtained from a rectangle by stretching, squeezing, and bending but without tearing or gluing together. Given a parametric representation $\mathbf{r} = \mathbf{r}(u,v)$ on a local patch, the values of the parameters u and v determine the position of each point on that patch. Construction and implementation of semigeodesic coordinates in our technique is described in [4].

2.1 Geodesic Lines

A geodesic line is defined as a contour which locally represents the shortest distance on a 3-D surface between any two points on that contour. Initially a geodesic line is drawn arbitrarily through the origin at the local area. This geodesic line is sampled at equal-sized intervals based on the average length of triangle edges. The second family of lines are also geodesic lines. All the lines together form semigeodesic coordinates.

Before semigeodesic coordinates can be generated on a local patch at a chosen vertex V, an arbitrary geodesic line is required. The edge connecting V and one of its neighbouring vertices is selected as the arbitrary direction. Once this direction is determined, the next step is to construct a geodesic line. This line is constructed on the local 3-D surface by following the geodesic path in a straight line until an edge or vertex is reached. This new edge point or vertex becomes a starting point for the next extension. To continue a geodesic line into the next triangle, we first measure the angle between the path and the common edge which the path has intersected. We then extend the path to the next triangle using the same angle. The construction of this geodesic line continues until last edge or vertex of local area is reached. Same process is repeated to construct the reverse direction of the geodesic line.

The second family of lines are constructed perpendicularly to the above newly created geodesic line. Each sampled point of that geodesic line is used as a reference point for constructing these perpendicular geodesic lines. The perpendicular lines are constructed in the forward direction as well as the backward direction with respect to the geodesic line. This completes the construction of local semigeodesic parametrisation. Semigeodesic coordinates can also be constructed at or near a boundary in case of an incomplete surface or a surface with holes. In such cases, geodesic lines are constructed as before but they are terminated as soon as they intersect the surface boundary.

2.2 2-D Gaussian Convolution

Gaussian filtering is a weighted average smoothing carried out at a vertex and its neighbourhood. The result of smoothing depends entirely on a vertex and its neighbourhood. So the filtering uses a *local area* of the surface with its size same as filter size. A large 3-D triangulated surface is overlaid by many fixed size small local areas. In order to eliminate over-sampling and under-sampling, the area size must be neither too large nor too small. In other words, a local area must cover a reasonable size neighbourhood in order to provide accurate results. Experiments were conducted with a filter size of 9 (optimal filter size) with $\sigma = 1.0$. In order to smooth a 3-D surface, a fixed size 2-D Gaussian filter with $\sigma = 1.0$ is convolved with the local area. Local parametrisation of the surface yields:

$$r(u, v) = (x(u, v),\ y(u, v),\ z(u, v))$$

The smooth surface is defined by:

$$R(u, v, \sigma) = (\mathcal{X}(u, v, \sigma),\ \mathcal{Y}(u, v, \sigma),\ \mathcal{Z}(u, v, \sigma))$$

where

$$\mathcal{X}(u, v, \sigma) = x(u, v)\ *\ G(u, v, \sigma)$$
$$\mathcal{Y}(u, v, \sigma) = y(u, v)\ *\ G(u, v, \sigma)$$
$$\mathcal{Z}(u, v, \sigma) = z(u, v)\ *\ G(u, v, \sigma)$$

and * denotes convolution. This process is repeated at each vertex, and the new vertex positions after filtering define the smoothed surface. This procedure is iterated several times to yield heat diffusion of the surface.

3 Curvature Estimation

This section presents techniques for accurate estimation of Gaussian and mean curvatures at multiple scales on smoothed free-form 3-D surfaces. Differential geometry provides several measures of curvature, which include Gaussian and mean curvatures [1]. Consider a local parametric representation of a 3-D surface $\mathbf{r}(u, v)$ with coordinates u and v, where

$$\mathbf{r}(u, v) = (x(u, v),\ y(u, v),\ z(u, v))$$

Gaussian curvature K exists at regular points of a surface of class C_2. When $\mathbf{r}(u, v)$ corresponds to semigeodesic coordinates, K is given by:

$$K = \frac{b_{uu} b_{vv} - {b_{uv}}^2}{x_v^2 + y_v^2 + z_v^2}$$

where subscripts denote partial derivatives, and

$$b_{uu} = \frac{A x_{uu} + B y_{uu} + C z_{uu}}{\sqrt{A^2 + B^2 + C^2}}$$

$$b_{vv} = \frac{A x_{vv} + B y_{vv} + C z_{vv}}{\sqrt{A^2 + B^2 + C^2}}$$

$$b_{uv} = \frac{A x_{uv} + B y_{uv} + C z_{uv}}{\sqrt{A^2 + B^2 + C^2}}$$

where

$$A = y_u z_v - z_u y_v$$

$$B = x_v z_u - z_v x_u$$

$$C = x_u y_v - y_u x_v$$

Mean curvature H also exists at regular points of a surface of class C_2. Again, when $\mathbf{r}(u, v)$ corresponds to semigeodesic coordinates, H is given by:

$$H = \frac{b_{vv} + (x_v^2 + y_v^2 + z_v^2) b_{uu}}{2(x_v^2 + y_v^2 + z_v^2)}$$

Both Gaussian and mean curvature values are direction-free quantities. Gaussian and mean curvatures are invariant to arbitrary transformations of the (u,v) parameters as well as rotations and translations of a surface.

3.1 Curvature 0-crossing Contours

Having computed curvature values at each vertex of a smoothed 3-D surface, one can locate curvature zero-crossing contours where curvature functions K or H are equal to zero. Curvature zero-crossing contours can be useful for segmenting a smoothed 3-D surface into regions. The process of recovery of the curvature zero-crossing contours is identical for Gaussian and mean curvatures. Every edge e of the smoothed surface is examined in turn. If the vertices of e have the same signs of curvature, then there is no curvature zero-crossing point on e. However, if the vertices of e have different signs of curvature, then there exists a point on e at which curvature goes to zero. The zero-crossing point is assumed to be at the midpoint of e. The other two edges of the triangle to which e belongs will then be checked since there will be another zero-crossing point on one of those edges. When that zero-crossing is found, it is connected to the previously found zero-crossing. The curvature zero-crossing contour is tracked in this fashion until one arrives back at the starting point.

3.2 Local Curvature Maxima

Local maxima of Gaussian and mean curvatures are significant and robust feature points on smoothed surfaces since noise has been eliminated from those surfaces. The process of recovery of the local maxima is identical for Gaussian and mean curvatures. Every vertex V of the smoothed surface is examined in turn. The neighbours of V are defined as vertices which are connected to V by an edge. If the curvature value of V is higher than the curvature values of all its neighbours, V is marked as a local maximum of curvature. Curvature maxima can be utilised by later processes for robust surface matching and object recognition with occlusion.

3.3 Maxima of Torsion of Curvature Zero-Crossing Contours

This section briefly reviews the computation of torsion. Torsion is the instantaneous rate of change of the osculating plane with respect to the arc length parameter. The osculating plane at a point P is defined to be the plane with the highest order of contact with the curve at P. Intuitively, torsion is a local measure of the nonplanarity of a space curve [1]. The set of points of a space curve are the values of a continuous, vector-valued, locally one-to-one function

$$r(u) = (x(u),\ y(u),\ z(u))$$

where $x(u)$, $y(u)$ and $z(u)$ are the components of $r(u)$, and u is a function of arc length of the curve. In order to compute torsion τ at each point of the curve, it is then expressed in terms of the derivatives of $x(u)$, $y(u)$ and $z(u)$. In case of an arbitrary parametrisation, torsion is given by,

$$\tau = \frac{\dot{x}(\ddot{y}\check{z} - \ddot{z}\check{y}) - \dot{y}(\ddot{x}\check{z} - \ddot{z}\check{x}) + \dot{z}(\ddot{x}\check{y} - \ddot{y}\check{x})}{(\dot{y}\ddot{z} - \dot{z}\ddot{y})^2 + (\dot{z}\ddot{x} - \dot{x}\ddot{z})^2 + (\dot{x}\ddot{y} - \dot{y}\ddot{x})^2}$$

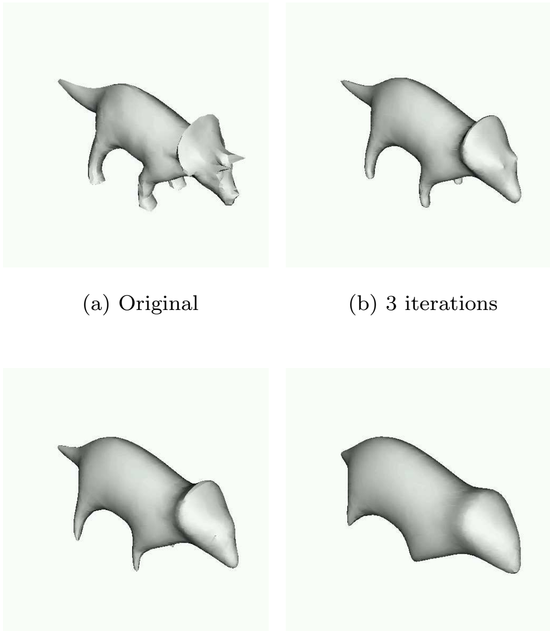
where $\dot{x}(u)$, $\dot{y}(u)$ and $\dot{z}(u)$ are the convolutions of $x(u)$, $y(u)$ and $z(u)$ with the first derivative of a 1-D Gaussian function. Note that $\ddot{x}$ and $\check{x}$ represent convolutions with the second and third derivatives of a 1-D Gaussian.

4 Results and Discussion

This section presents some results on free-form surface smoothing as well as curvature estimation.

4.1 Diffusion

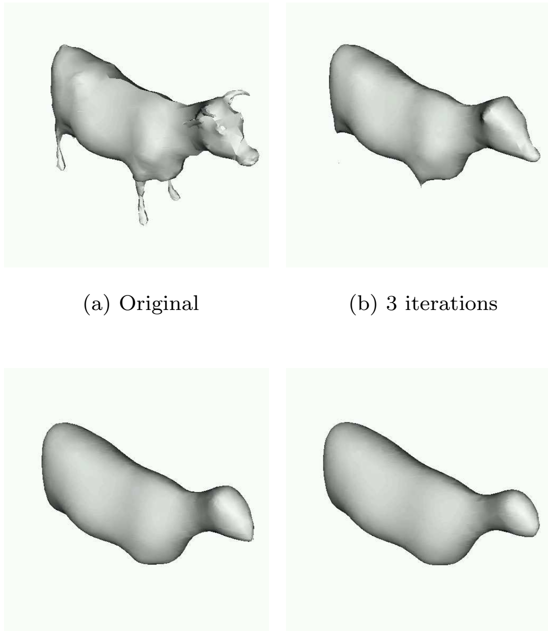
The smoothing routines were implemented entirely in C++. Each iteration of smoothing of a surface with 1000 vertices takes about 0.5 second of CPU time on an UltraSparc 170E. The first test object was a dinosaur with 2996 triangles and 1500 vertices as shown in Figure 1. The object becomes smoother



(a) Original (b) 3 iterations

(c) 5 iterations (d) 10 iterations

Figure 1: Smoothing of the dinosaur



(a) Original (b) 3 iterations

(c) 8 iterations (d) 12 iterations

Figure 2: Smoothing of the cow

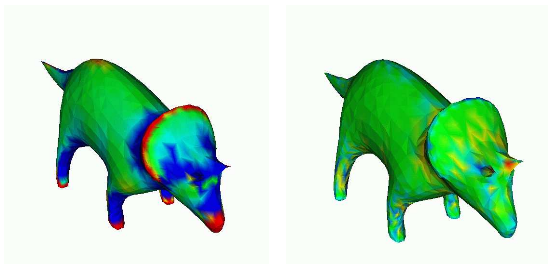
gradually and the legs, tail and ears are removed after 10 iterations. The second test object was a cow with 3348 triangles and 1676 vertices as shown in Figure 2. The surface noise is eliminated iteratively with the object becoming smoother gradually where after 12 iterations the legs, ears and tail are removed, as was seen for the dinosaur. These examples show that our technique is effective in eliminating surface noise as well as removing surface detail. The result is gradual simplification of object shape.

4.2 Curvature Estimation

This section presents the results of application of our curvature estimation techniques. To visualise these curvature values on the surface, they are then mapped to colours/grey-scales using the Visualisation Toolkit (VTK) [6]. Gaussian curvatures are shown in Figure 3(a). Surface curvature values are coded as follows: *bright = high, dark = low and other grey-scales designate in-between values.* All convex corners of the dinosaur are bright, indicating high curvature values, whereas the concave corners are dark indicating low curvature values and flat areas are grey since their curvature values are close to zero. The same experiment was repeated to estimate the mean curvatures of the dinosaur and the results are shown in Figure 3(b). This indicates that mean curvature values for the edges are different than those for flat areas, as expected. The Gaussian and mean curvatures were also estimated for the cow, as shown in Figure 4.

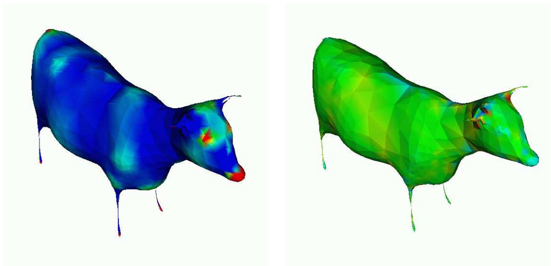
Next, the curvature zero-crossing contours of these surfaces were found and displayed on the surface using VTK. Curvature zero-crossing contours can be used for segmenting surfaces into regions. Figures 5(a) and (b) shows Gaussian curvature zero-crossing contours for the smoothed dinosaur. Figures 5(c) and (d) shows mean curvature zero-crossing contours for the same object. The same experiments were repeated for cow, and the results are shown in Figure 6. Notice that the number of curvature zero-crossing contours are reduced, as the object is further smoothed.

Next, the local curvature maxima for the



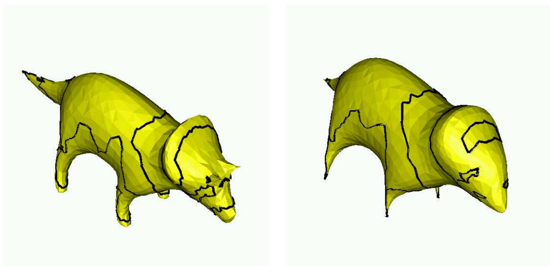
(a) Gaussian (b) Mean

Figure 3: Curvature values on the dinosaur

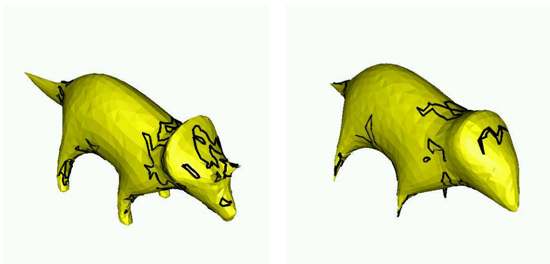


(a) Gaussian (b) Mean

Figure 4: Curvature values on the cow

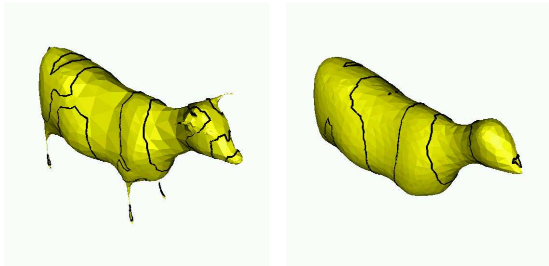


(a) One iteration (b) 6 iterations

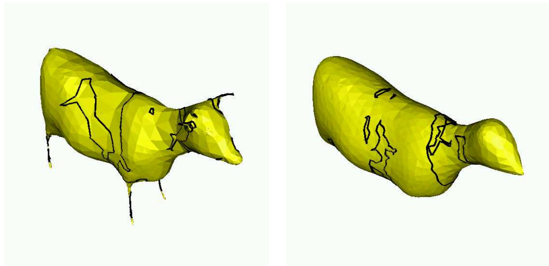


(c) One iteration (d) 6 iterations

Figure 5: Gaussian (top row) and mean (bottom row) curvature zero-crossing contours on dinosaur



(a) One iteration (b) 6 iterations

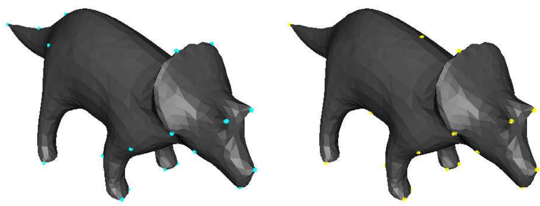


(c) One iteration (d) 6 iterations

Figure 6: Gaussian (top row) and mean (bottom row) curvature zero-crossing contours on the cow

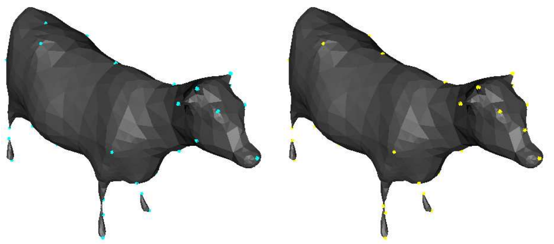
smoothed dinosaur were computed. The local maxima of Gaussian curvature are displayed on the surface as shown in Figure 7(a). Figure 7(b) shows the local maxima of mean curvature for the same object. The local maxima of Gaussian and mean curvatures for the cow are shown in Figure 8. All curvature maxima are shown after one iteration.

Finally, the torsion maxima of curvature zero-crossing contours which are alternative features that can be used for matching are determined and displayed on the object. Figures 9(a) and (b) show the torsion maxima of curvature zero-crossing contours of the dinosaur for Gaussian and mean curva-



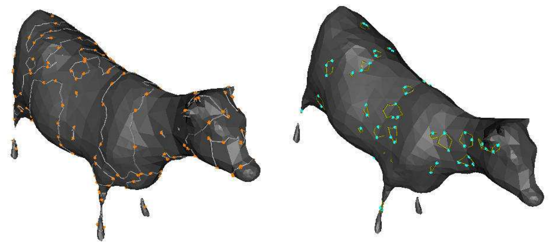
(a) Gaussian (b) Mean

Figure 7: Curvature maxima of the dinosaur



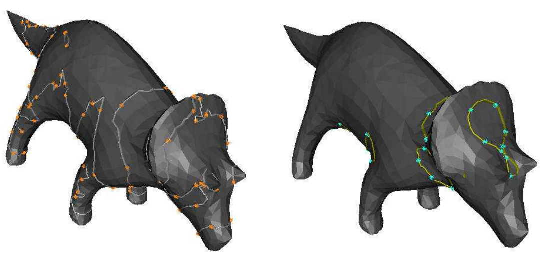
(a) Gaussian (b) Mean

Figure 8: Curvature maxima of the cow



(a) Gaussian (b) Mean

Figure 10: Torsion maxima of curvature zero-crossing contours of the cow



(a) Gaussian (b) Mean

Figure 9: Torsion maxima of curvature zero-crossing contours of the dinosaur

tures, respectively. Figure 10 shows the results for the cow.

These features can be utilised by later processes for robust surface matching and object recognition with occlusion. Animation of surface diffusion can be observed at the web site: http://www.ee.surrey.ac.uk/Research/VSSP/demos/css3d/

5 Conclusions

A novel technique for multi-scale curvature computation on a smoothed 3-D surface is presented. In our technique semigeodesic coordinates are constructed at each vertex of the mesh which becomes the local origin. A geodesic from the origin is first constructed in an arbitrary direction such as the direction of one of the incident edges. During the diffusion process, 3D surfaces were also sampled locally using different step sizes. Complete triangulated models of 3-D objects are constructed and using a local parametrisation technique, are then smoothed using a 2-D Gaussian filter. The smoothing eliminated the surface noise and small surface detail gradually, and resulted in gradual simplification of object shape. The surface Gaussian and mean curvatures were also estimated. To visualise these curvature values on the surface, they are then mapped to colours/grey-scales, and shown directly on the surface using the Visualisation Toolkit. All convex corners of the surface indicated high Gaussian curvature values, whereas the concave corners indicated low Gaussian curvature values and the curvature values of flat areas are close to zero.

Gaussian and mean curvature zero-crossing contours were also recovered and displayed on the surface. Results indicated that as the surface is smoothed iteratively, the number of curvature zero-crossing contours were reduced. Curvature zero-crossing contours can be used for segmenting surfaces into regions. Furthermore, the local maxima of Gaussian and mean curvatures as well as the torsion maxima of zero-crossing contours of Gaussian and mean curvatures were also located and displayed on the surface. These features can be utilised by later processes for robust surface matching and object recognition with occlusion.

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